Optimal Control of an Electromechanical Above-Knee Prosthesis with Energy Regeneration

Farbod Rohani, Hanz Richter, Antonie J. van den Bogert
Parker Hannifin Laboratory for Human Motion and Control (hmc.csuohio.edu)
Department of Mechanical Engineering, Cleveland State University

Introduction

During walking, there are periods in the gait cycle that knee and/or ankle produce negative work [1]. Here we present a concept for a powered prosthetic leg for transfemoral amputees that can store and re-use this energy.

The proposed system consists of two motors, two four-quadrant power converters, two springs, and an ultracapacitor for energy storage (Figure 1).

Objectives:

- Determine optimal design parameters
- Determine the trade-off between energy use and performance
- Determine performance of the system when used without external energy in different subjects and speeds

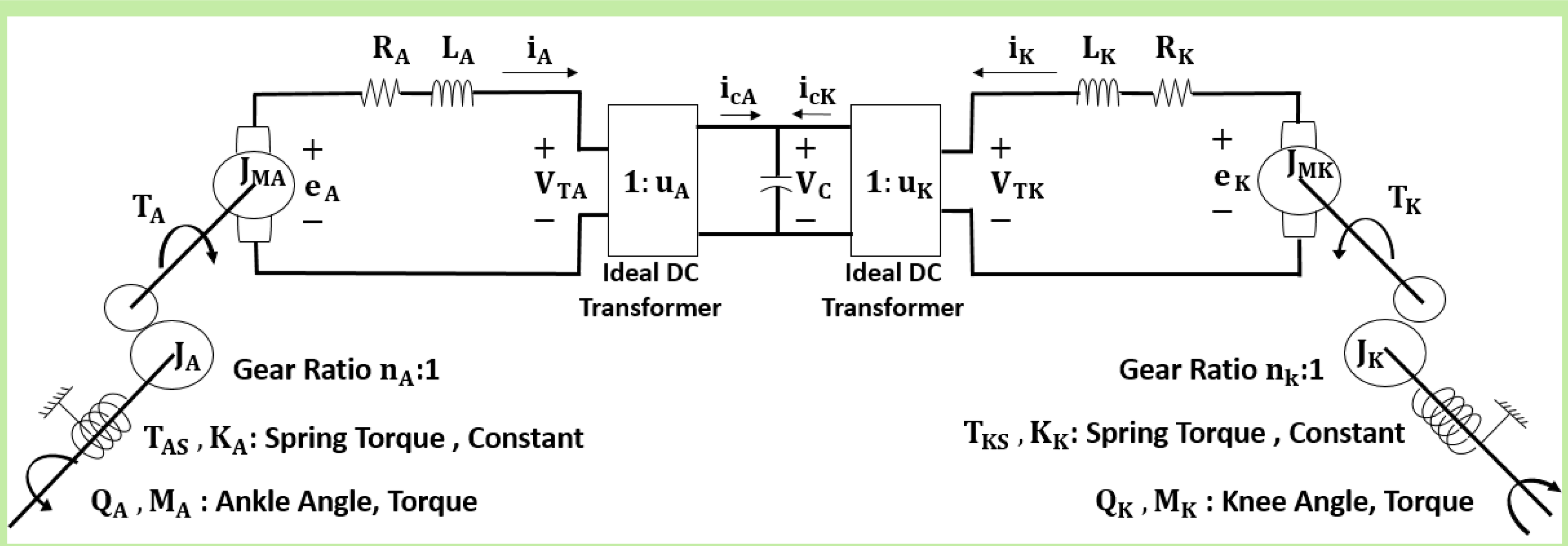

Fig 1. Schematic diagram of the electromechanical prosthetic system with motors at ankle (left) and knee (right).

Methods

System Dynamics:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{K_K}{J_{TK}}x_1 - \left(\frac{a_K^2 n_K^2}{R_K J_{TK}} + \frac{b_K}{J_{TK}}\right)x_2 + \frac{a_K n_K U_K(t)}{R_K\, C J_{TK}}x_3 + \frac{M_K(t)}{J_{TK}}$$

$$\dot{x}_3 = \frac{a_K n_K U_K(t)}{R_K}x_2 - \left(\frac{U_K^2(t)}{R_K C} + \frac{U_A^2(t)}{R_A\, C}\right)x_3 + \frac{a_A n_A \dot{U}_A(t)}{R_A}x_5$$

$$\dot{x}_4 = x_5$$

$$\dot{x}_5 = -\frac{K_A}{J_{TA}}x_4 - \left(\frac{a_A^2 n_A^2}{R_A J_{TA}} + \frac{b_A}{J_{TA}}\right)x_5 + \frac{a_A n_A U_A(t)}{R_A\, C J_{TA}}x_3 + \frac{M_A(t)}{J_{TA}}$$

Optimal Control Problem:

- Inputs: knee and ankle torques from gait data [3]
- Find periodic state and control trajectories $x(t)$ and $u(t)$, and design parameters n_K, n_A, K_K, K_A that satisfy the system dynamics and minimize a weighted sum of tracking error and energy loss:

$$F = W_1\int_0^T \left[\left((\theta_K(t) - \theta_{K,0}(t))/\sigma_K\right)^2 + \left((\theta_A(t) - \theta_{A,0}(t))/\sigma_A\right)^2\right]dt + W_2\frac{Q(0)^2 - Q(T)^2}{2C}$$

- Solved by Direct Collocation [2]

Protocol:

- Data from 10 subjects walking at 0.8, 1.2, and 1.6 m/s
- Pareto analysis in one subject walking at 1.2 m/s
- Zero-energy analysis in all subjects, all speeds
- 24V DC motor (Pittman, 14201 series)

Results

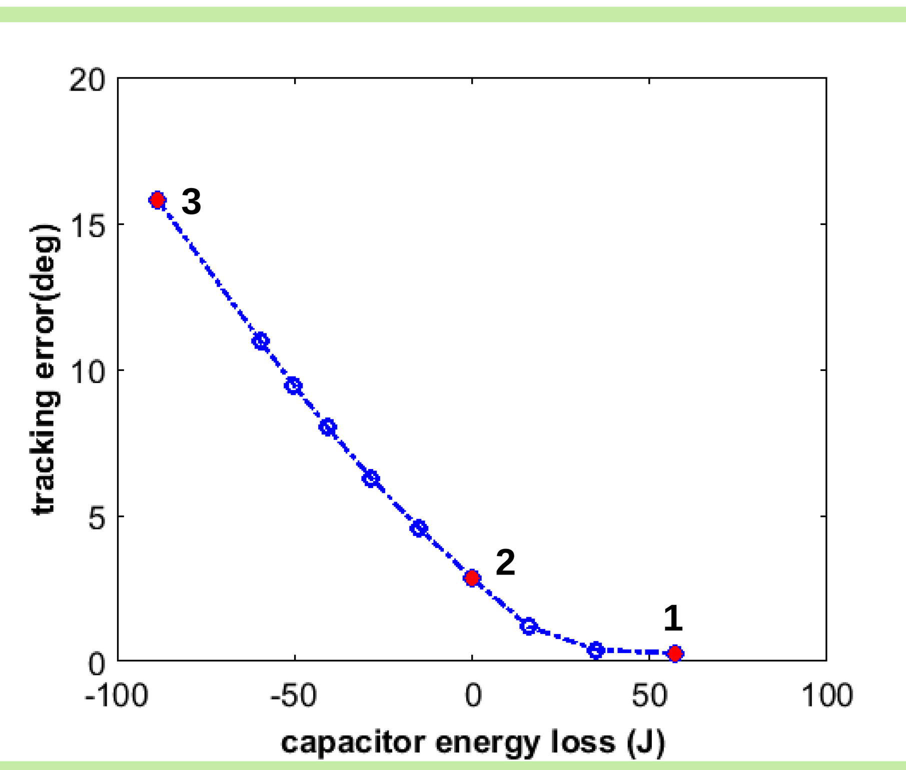

Fig 2. The entire range of possible performances is shown as a Pareto front

- Pareto optimizations show the trade-off between the tracking objective and the capacitor energy loss (Figure 2, Table 1)
- Case 1: Almost perfect tracking and energy loss of 57.7 J
- Case 2: No energy loss, RMS tracking error 2.8° which is an acceptable performance. Fig. 3 shows the system trajectories.
- Case 3: More than enough energy is harvested to operate the system, but the tracking error of 16° is unacceptably large.

Table 1 Work-energy values, optimized gear ratios and optimized spring constants for the three Pareto-optimal solutions at 1.2 m/s.

Case Number	W_{knee} (J)	W_{ankle} (J)	Heat (J)	ΔE_{cap} (J)	n_{knee}	n_{ankle}	K_{knee} (Nm/rad)	K_{ankle} (Nm/rad)
1	-6.21	4.90	58.38	-57.07	123.08	158.76	1.75	29.64
2	-21.04	-1.53	22.48	0	220.58	257.12	20.78	330.79
3	-91.23	-28.38	30.33	88.62	148.13	242.03	23.43	322.26

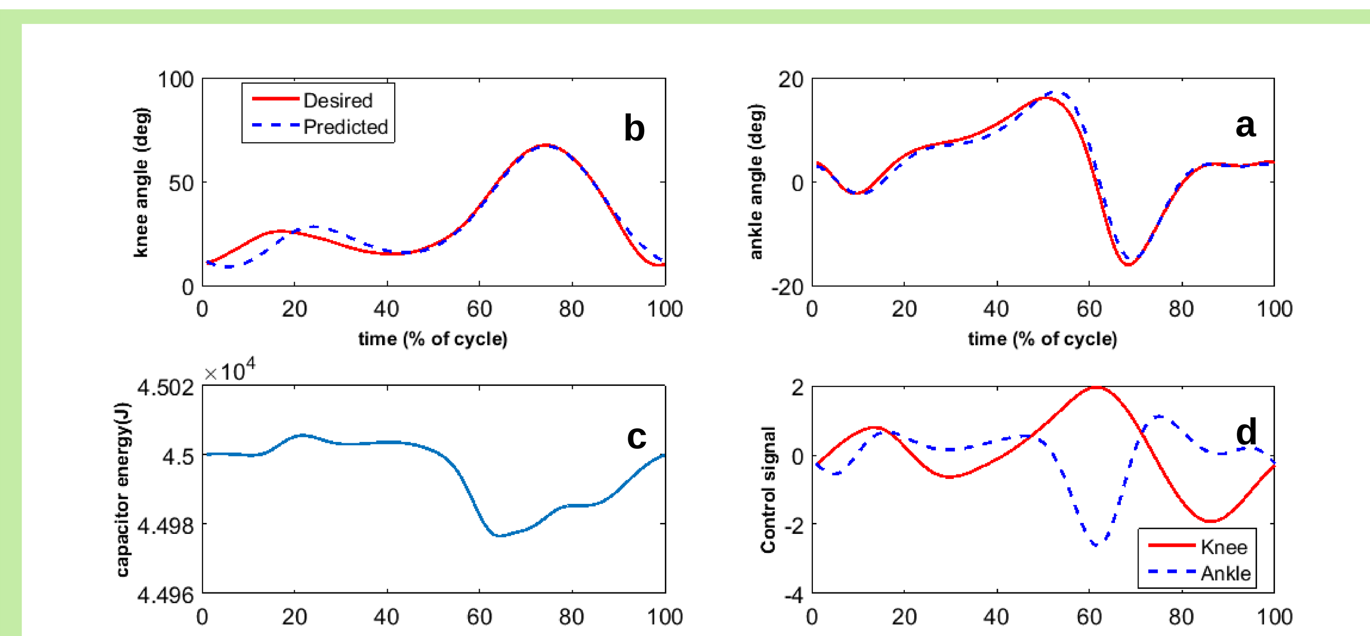

Fig 3. Joint angles (a,b) and capacitor energy (c) and control signals (d) for a typical subject at 1.2 m/s without energy loss.

Table 2 The mean and standard deviation of optimized parameters in ten subjects walking with different speeds [3]

	Slow walking	Normal walking	Fast walking
RMS error*(deg)*	3.0 ± 1.3	4.3 ± 1.4	5.6 ± 1.4
W_{knee}(J)	-10.3 ± 2.7	-28.0 ± 6.1	-49.2 ± 12.9
W_{ankle}(J)	-6.4 ± 4.4	-4.8 ± 3.0	-2.75 ± 3.1
W_{heat}(J)	16.6 ± 3.9	32.6 ± 5.9	51.8 ± 11.1
k_{knee}(Nm/rad)	14.9 ± 6.4	24.7 ± 9.5	34.6 ± 13.2
k_{ankle}(Nm/rad)	445 ± 110	458 ± 124	495 ± 118
n_{knee}	243 ± 38	231 ± 34	227 ± 38
n_{ankle}	381 ± 60	332 ± 40	332 ± 47

- With increasing speed, the tracking error increases as well as the energy dissipated as heat in the motor resistor.
- Optimal design parameters are speed-dependent

Conclusion

✔ A powered knee-ankle prosthesis can operate without external energy and with acceptable trajectory error for the joints.

✔ By allowing more error in knee/ankle position, more energy can be harvested from the system.

References

1. Winter DA. *Clin Orthop Rel Res* 175, 147-154, 1983.
2. van den Bogert AJ, et al. J Biomech Eng 134, 051007, 2012.
3. Moore J, et al. *PeerJ 3* (2015): e918.

Acknowledgments: supported by the National Science Foundation under grants No. 1344954 and 1544702.