# Trajectory Optimization in Stochastic Multibody Systems using Direct Collocation

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Recent studies have attempted to find predictive simulations of human walking [1] and running [2]. These models can predict the main features of human movement, but not all. This is partly due to modeling assumptions and the selection of optimization criteria, but a more fundamental limitation is that the models are deterministic and noise in the human system or in the mechanical environment is not taken into account. Recent studies suggest that this noise is important to explain certain human movement strategies [3, 4]. Our long term goal is to perform predictive simulations of human movement using stochastic dynamics.

Predictive simulations of human movement are formulated as trajectory optimization problems, similar to robotics and aerospace applications. In practice, noise in the system is often ignored and the problem is solved with a deterministic model. This produces a correct solution for the stochastic case when the system is linear with a quadratic cost function. However, human gait has nonlinear dynamics, with limbs alternating between stable pendulum in swing and inverted pendulum in stance. With these dynamics, a true stochastic trajectory optimization must be performed. This is a hard problem which has only been solved in low-dimensional systems and some special cases (e.g. [5]).

Here we propose a new approach to optimize a trajectory in a stochastic environment using direct collocation. To prove the concept, we consider a simple pendulum swing-up problem with a simple controller. The pendulum nonlinearities are qualitatively similar to those of human gait, and we expect that the optimal trajectory will be different between a stochastic environment and a deterministic environment. We will show that our method can solve the stochastic optimal control problem, and that the solution depends on the magnitude of the noise.

The pendulum has one degree-of-freedom, the angle between the ground and the pendulum, $\theta$. The system thus has two states, the angle and the angular velocity, so $x = [\theta \quad \dot{\theta}]^T$. The torque at the base, $u$, controls the pendulum. This yields the following dynamics:

$$\dot{x}(t) = \begin{bmatrix} \dot{\theta}(t) \\ -\frac{g}{l} cos(\theta(t)) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{u(t)}{ml^2} \end{bmatrix} + \begin{bmatrix} 0 \\ N\ (0, \sigma^2) \end{bmatrix} \tag{1}$$

where $g$ denotes gravity, $m$ and $l$ are the mass and length of the pendulum, 5 kg and 1.2 m, respectively. The system is deterministic if the variance, $\sigma^2$, equals 0 and stochastic for any nonzero variance.

The aim is to optimize a trajectory that swings up the pendulum from a downward position ($\theta = -\pi/2$) to an upward position ($\theta = \pi/2$) in 10 seconds, while minimizing the squared torque. Solving this trajectory optimization problem in a deterministic environment is straightforward and can be done with open loop control $u(t)$. However, feedback is required in a stochastic environment to handle all possible instances of the noise. We propose to optimize the task on samples of the problem, each representing a different instance of the noise and producing a different trajectory, but with the same controller. Linear feedback is assumed: $u(t) = f(t) + Kx(t)$. The time-varying term, $f(t)$, and the 1$x$2 gain matrix K, should be the same in all samples of the problem.

A direct collocation approach was used, with the trajectory represented by the state $x$ at $N$ collocation points. The optimization is carried out over $M$ trajectory samples, each with a different instance of the noise term in the dynamics. The open-loop control term $f(t)$ is discretized over the $N$ collocation points. Similar to human gait, we do not require that each of the $M$ trajectories ends in the desired final state, but rather that the average trajectory satisfies the task. This yields a large scale non-linear programming (NLP) problem with $2NM + N + 2$ unknowns: $M$ state trajectories and the open loop control, each represented by $N$ collocation points, and the two feedback gains. The objective $J(x,u)$ and task constraints $g(x)$ are formulated as:

$$J(x,u) = \frac{1}{2M}\sum_{j=1}^{M}\sum_{i=1}^{N} u_{ij}^2 \quad (2)$$

$$g(x) \equiv \frac{1}{M}\sum_{j=1}^{M} x_{ij} - \begin{bmatrix}\frac{\pi}{2}\\ 0\end{bmatrix} = 0 \quad (3)$$

Additionally, $M(N-1)$ dynamics constraints were formulated by applying the Backward Euler discretization formula [1] to the dynamic model (1). The optimization was done with $N = 60$. The number of samples was increased until there was no longer a significant change in average optimal trajectory. This occurred at $M = 20$.

Fig. 1 shows the optimal trajectories that are found with different noise levels. In the deterministic case ($\sigma = 0$), the swing up takes full advantage of passive dynamics. The energy is gradually increased through countermovements, and gravity slows down the pendulum to make it stop exactly in the upright position. In the presence of noise, it is not possible to achieve the desired final state in each trial, but it is achieved on average. With increasing noise levels, the optimal trajectory changes. The final swing-up motion to an upright position becomes more and more delayed. This happens because in a noisy environment, it becomes costly to spend more time near the upright, unstable equilibrium.

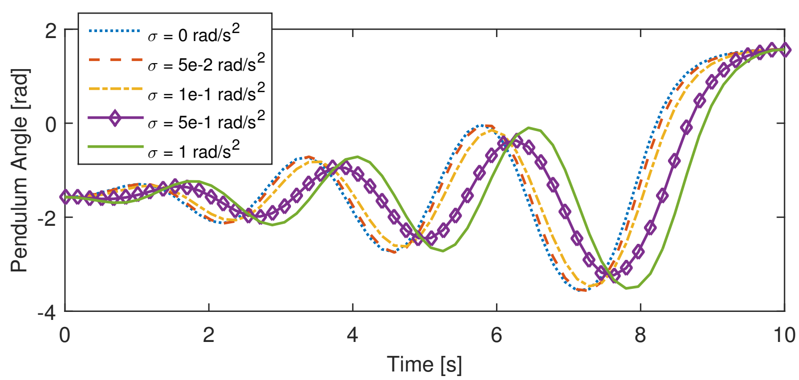


Fig. 1: Optimal trajectories that were found with different noise levels. For the stochastic cases ($\sigma > 0$), the average of all trajectory samples is shown.

In conclusion, we showed that the proposed trajectory optimization method is able to find an optimal trajectory for a stochastic nonlinear system, and that the optimal trajectory depends on the magnitude of the noise. Future work should expand the method to more complex multibody systems and control methods. Finally, we aim to use this method to find predictive simulations of gait. Issues might arise due to the high number of degrees of problem that this problem has and due to scaling in the number of decision variables and the required number of samples.

## References

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