A Novel Approach to Solve Predictive Simulations in a Stochastic Environment

Anne D. Koelewijn and Antonie J. van den Bogert
Parker Hannifin Laboratory for Human Motion and Control
Department of Mechanical Engineering, Cleveland State University

Introduction

- **Deterministic Models** cannot predict gait sufficiently [1]
- Noise is important for certain human movement strategies [2,3]
- Long term goal: predictive simulations of human movement using stochastic dynamics.

Goals of this study:
Propose method to find an optimal trajectory in a stochastic environment
1. Show that this method finds a different optimal trajectory in a stochastic environment
2. Show that muscle co-contraction minimize effort in certain tasks in a stochastic environment

Methods

Proposed Stochastic Optimization Approach

Minimize $\frac{1}{M}\sum_{j=1}^{M} J_j(x,u)$ Objective

Subject to $\dot{x} = f(x,u)$ Dynamics Constraints

$g_h(x) = 0$ Task Constraints

Variable	Meaning
i	Collocation Point
j	Episode
k	Muscle
N	Number of Collocation Points
M	Number of Episodes
T	Torque
u	Input (Torque or Muscular Activation)
K	Feedback Gain
x	State

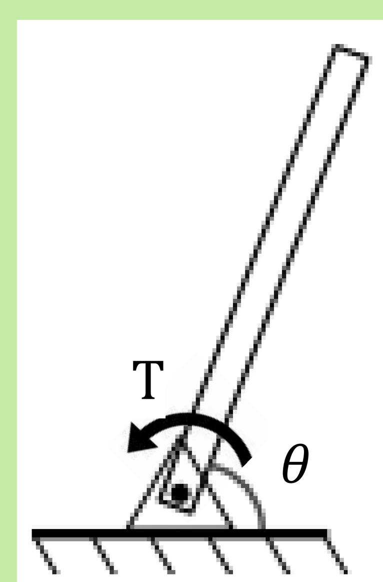
- Dynamics constraints: Direct collocation with backward Euler formulation
- Task constraint depends on problem
  - Average over number of episodes
  - Periodicity constraint
- Requires Feedback Control

Verification Using Pendulum
A one degree of freedom pendulum is used to verify the proposed stochastic optimization method.

Pendulum Dynamics:

$$x = [\theta \quad \dot{\theta}]^T$$

$$\dot{x} = \begin{bmatrix} \dot{\theta} \\ -\frac{mgl}{J}cos(\theta) + \frac{T}{J} \end{bmatrix} + \varepsilon \sim N(0,\sigma^2)$$



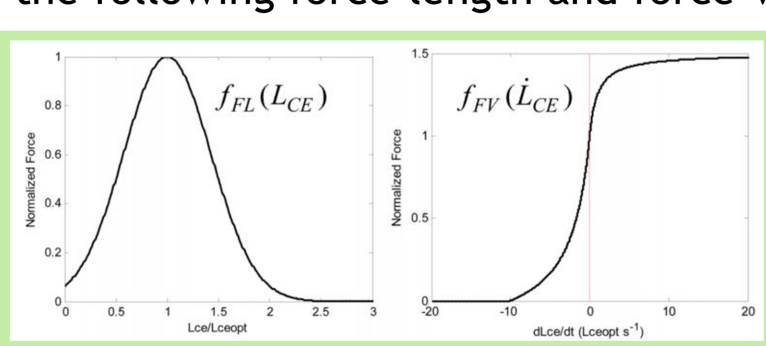
Pendulum

Muscles
Two similar Hill-type muscles are used for goal 2:

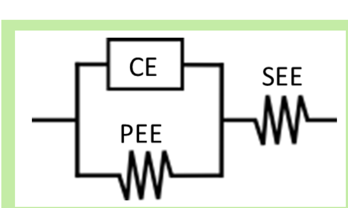
$$x = [\theta \quad \dot{\theta} \quad l_{CE_1} \quad l_{CE_2} \quad a_1 \quad a_2]^T$$

$$F_{CE} = a(u)F_{max}f_{FL}(l_{CE})f_{FV}(\dot{l}_{CE})$$

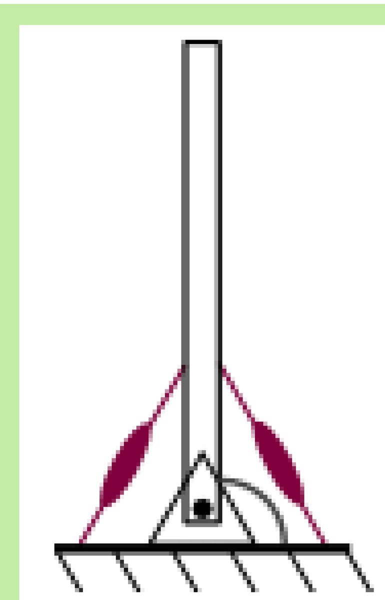
with the following force-length and force-velocity relationships



Force-Length and Force-Velocity Relationships



Hill-Type Muscle


Pendulum with muscles

Results

Pendulum Swing-Up

Objective: average minimal torque

$$J(x,u) = \frac{1}{2M}\sum_{j=1}^{M}\sum_{i=1}^{N} u_j(i)^2$$

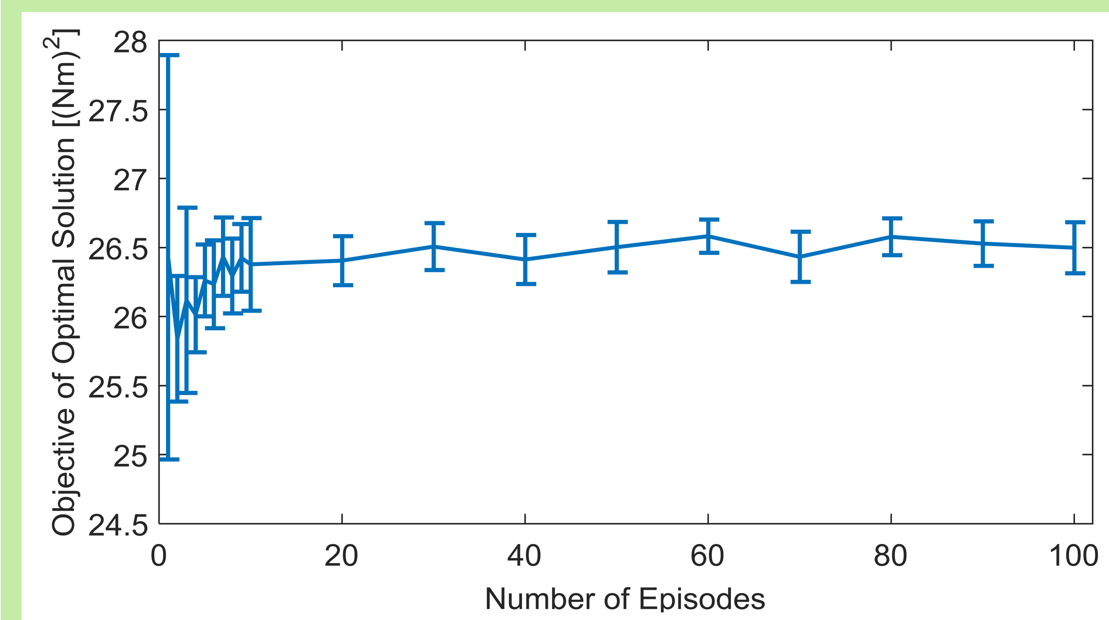
Task constraints: average trajectory of all episodes is a swing-up

$$x(0) + \begin{bmatrix} \frac{\pi}{2} \\ 0 \end{bmatrix} = 0, \qquad \frac{1}{M}\sum_{j=1}^{M} x(N) - \begin{bmatrix} \frac{\pi}{2} \\ 0 \end{bmatrix} = 0$$

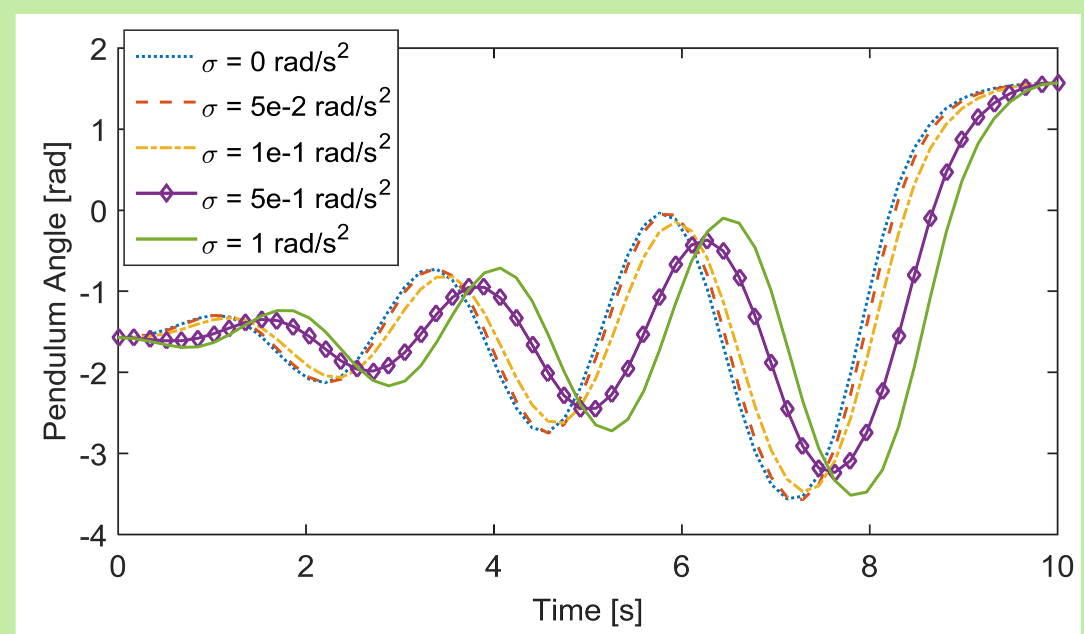
- Find required number of episodes
- Optimize trajectory in deterministic and stochastic environment

The torque is the input:
$$T = u = u_0(t) + Kx(t)$$

Theoretically, the stochastic problem is solved as the number of episodes M goes to infinity. This figure shows that for this problem, $M = 20$ is sufficient. The mean and standard deviation (solving multiple instances of the stochastic problem) are no longer changing.



Average objective as a function of the number of episodes



Optimal trajectories with increasing standard deviation

The optimal trajectories are plotted for different magnitudes of the noise. With increasing noise, the swing-up occurs later. The pendulum avoids spending time in unstable postures ($\theta > 0$), which would be costly in a stochastic environment.

Co-Contraction

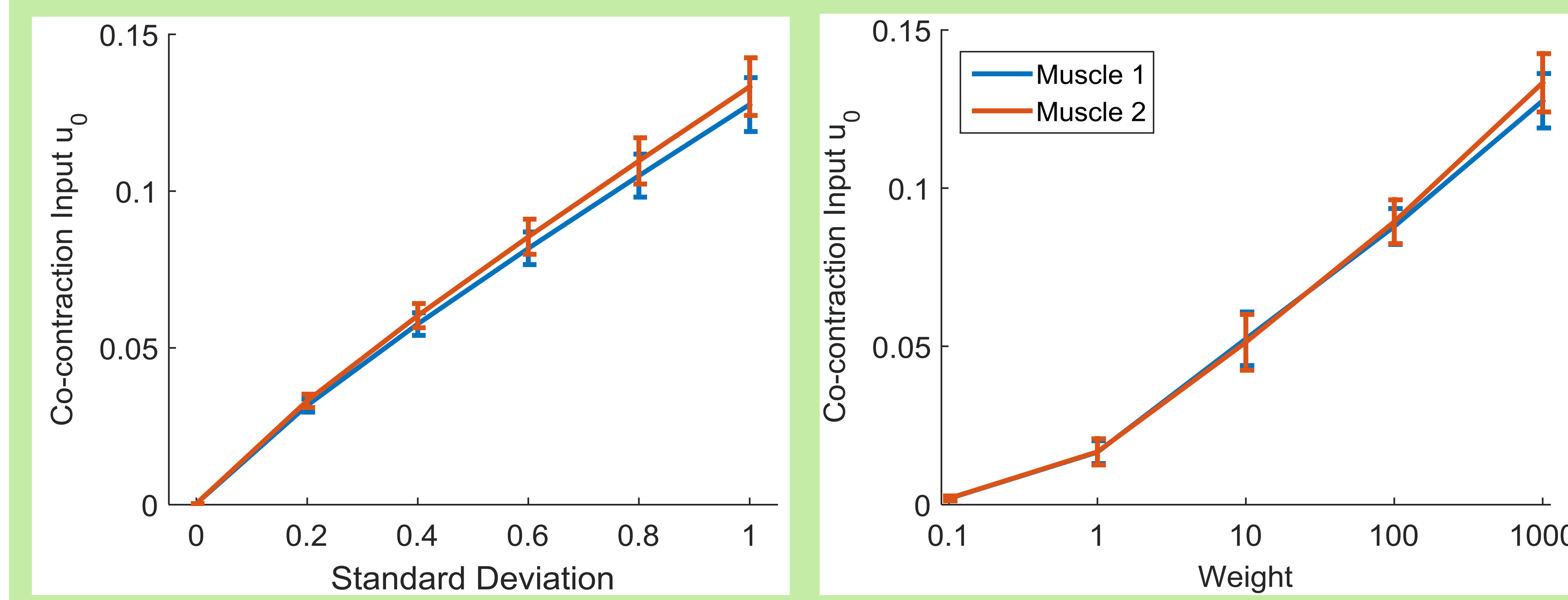
Objective: maintain upright position with minimal effort

$$J(x,u) = \sum_{i=1}^{N} W\left(\theta(i) - \frac{\pi}{2}\right)^2 + \sum_{k=1}^{2} u_k(i)^2$$

Task constraint: periodic motion
$$g(x) = x(1) - x(N+1) = 0$$

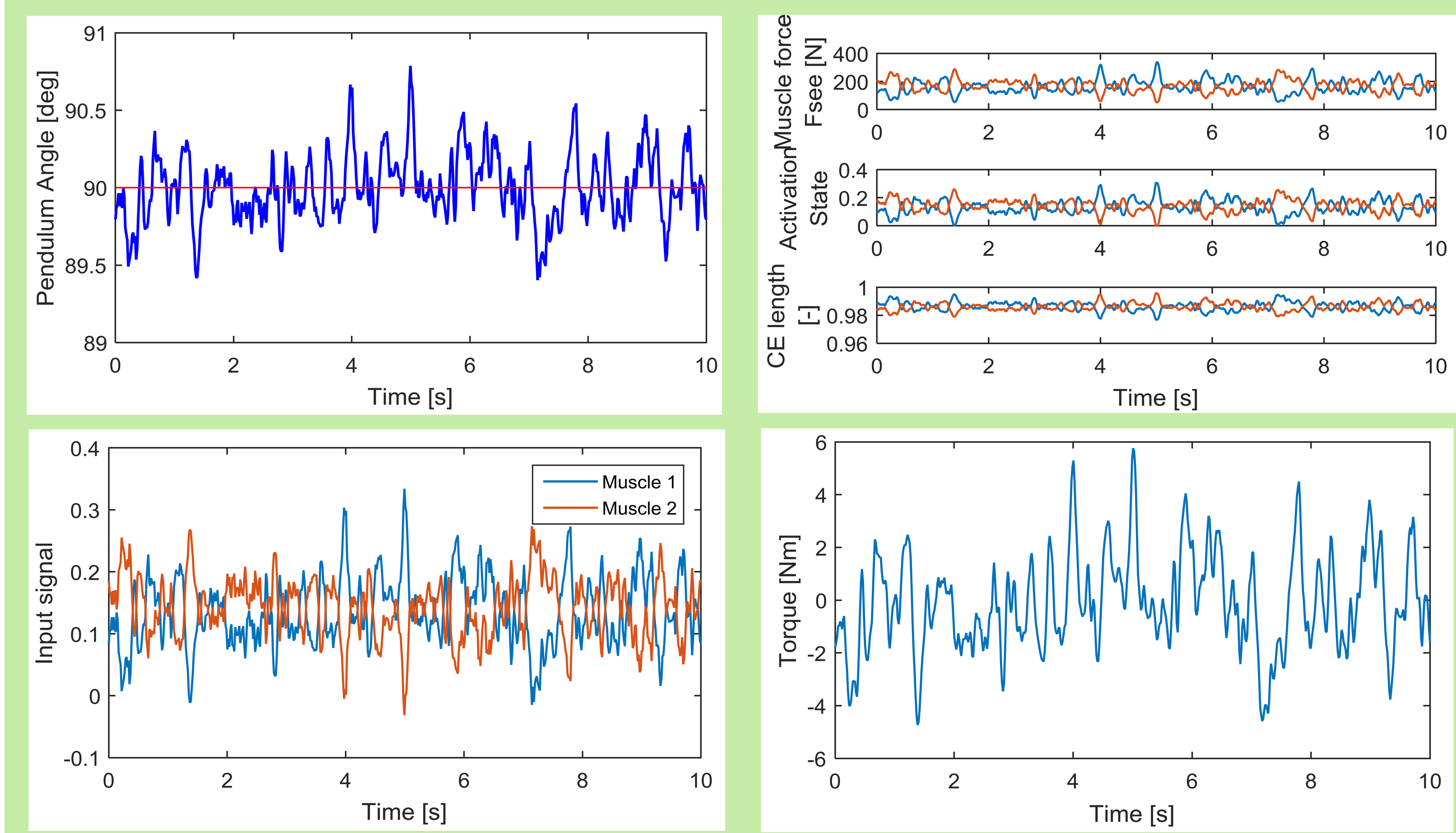
Show that co-contraction minimizes effort for certain tasks

For each muscle:
$$u_k = u_{0,k} + K_k x > 0$$
A nonzero u_0 in the muscles means that there is co-contraction



Amount of co-contraction in the muscles for different standard deviation of the noise and weight of the objective

Co-contraction requires less effort than only feedback in a task where the aim is to keep the pendulum in an upright position.



Typical Individual Result for $\sigma = 1$ and $W = 1000$

Conclusions

Successful verification of the proposed approach to solve predictive simulations in a stochastic environment:
- A different optimal trajectory was found in a stochastic environment than in a deterministic environment
- Co-contraction minimizes effort in tasks where increasing stiffness is less costly than compensating for errors

Implementation of approach on predictive simulations of gait.
- Improve predictions of normal walking
- Explain co-contraction reported in transtibial amputee gait [4]

References

[1] M. Ackermann, and A. J. van den Bogert (2010). J Biomech 43-6: 1055-1060.
[2] J. M. Donelan et al. (2004). J Biomech 37-6: 827-835.
[3] M. J. Hiley and M. R. Yeadon (2013). Hum Mov Sci 32-1: 181-191.
[4] E. Isakov et al. (2000). Prosthet Orthot Int 24-3: 216-220

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