Trajectory optimization for musculoskeletal state estimation with IMU sensors

Ton van den Bogert
Mechanical Engineering Department
Combination of two conference presentations

Optimal estimation in human movement from smoothing splines to musculoskeletal optimal control
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XVth International Symposium on 3D Analysis of Human Motion
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Herman J. Woltring memorial lecture

Trajectory Optimization for Human Motion Analysis based on Inertial Sensors
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International Society of Biomechanics
American Society of Biomechanics
Calgary, Canada, August 2019
Herman Woltring (1943-1992)

A FORTRAN PACKAGE FOR GENERALIZED, CROSS-VALIDATORY SPLINE SMOOTHING AND DIFFERENTIATION

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INTRODUCTION

Traditional studies of the kinematics of human movement have made extensive use of photographic and cinematographic data registration, followed by tedious and error-prone manual data-reduction procedures. Even though such data reduction is now becoming amenable to automation by the use of film scanners and pattern recognition algorithms, the delay between data acquisition and presentation exceeds clinically acceptable turnaround times and hinders real-time data processing for feedback

Based on the subroutine package developed at Risø, 

A LEAST-SQUARES ALGORITHM FOR THE EQUIFORM TRANSFORMATION FROM SPATIAL MARKER CO-ORDINATES

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ABSTRACT—The present paper describes an algorithm for estimating the translation vector and the rotation matrix of a moving body from noisy measurements on the spatial coordinates of at least three non-collinear

Optoelectric (Selspot) Gait Measurement in Two- and Three-Dimensional Space—A Preliminary Report

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1980

50

1989

ASE

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B I O M C H - L

An Electronic Mail Discussion List for Biomechanics and Kinesiology

Anton J. van den Bogert\textsuperscript{1} and Herman J. Waltring\textsuperscript{2}

An electronic distribution list has been created for members of the International Society of Biomechanics (ISB) and of related organizations (e.g., European, American, and Canadian Societies of Biomechanics) which, at least for users of EARN/BITNET/NETNORTH systems, allows free exchange of information with fellow-members on the list. In view of the overlap between Biomechanics and other fields such as Kinesiology, Bioengineering, Motor Control, and Physiology, the list is also open to non-members. At the time of writing (January, 1989), there are about 45 known subscribers in Belgium, Canada, Finland, France, Ireland, Italy, Netherlands, United Kingdom, and the United States, plus an unknown number of readers on Usenet, the news posting system for UUCP (Unix-to-Unix CoPy).

Activities on the list include discussions, congress reports, calls for help, calls for papers, and anything else relevant to the target domain. It is considered correct procedure that summaries of replies received in response to “calls for help” are posted for the benefit of all readers.

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A generalization of these approaches is to define a state-space model of rigid body positions, attitudes, and their (first and second) derivatives, and to apply optimal prediction, filtering, and smoothing techniques [...]. This has the additional advantages of simultaneous derivative estimation and of automatic interpolation in the event of partial or complete data loss [...]. Unfortunately, such more universal procedures are numerically expensive, and they require familiarity with contemporary developments in the realm of optimal control, system identification, and parameter estimation.
Optimal estimation

- Is important in human movement, because measurements are often:
  - inaccurate and noisy
  - incomplete
  - overcomplete & multiple modalities

- Models can help
  - (but aren't perfect...)

- Optimal estimation
  - can make use of all available measurements
  - optimally combines measurements and model, with knowledge of how much to trust each of them
PRINCIPLES

Rev. Thomas Bayes (1701-1761)
Sir Isaac Newton (1643-1726)
Bayesian inference

*E*: evidence (measurements)

*H*: hypothesis (estimates of variables of interest)

Bayes' theorem:

\[ P(H \mid E) \cdot P(E) = P(E \mid H) \cdot P(H) \]

\[ \Rightarrow \quad P(H \mid E) = \frac{P(E \mid H) \cdot P(H)}{P(E)} \]

Given the measurements *E*, which estimate *H* is best?

Answer: find the *H* for which \( P(H \mid E) \) is highest

\[ \hat{H} = \arg \max_H \left[ P(E \mid H) \cdot P(H) \right] \]
Two approaches

Recursive (real time)
- Kalman filters, particle filters
- uses past measurements only

Trajectory optimization (not real time)
- uses past and future measurements

Both use the same "ingredients"
- process model
- observation model
- assumptions about how much to trust these models
  - Kalman filters: Q and R covariance matrices
  - Trajectory optimization: effort weighting coefficient
Dynamic motion estimation

- Nobody believes that this is actual human motion
  - we know about Newton's laws of motion
  - we know there is measurement error
  - how do we estimate the actual motion?

marker coordinate from optical motion capture
Optimal dynamic motion estimation (1-dimensional)

Trajectory to be estimated: $x(t), \ 0 \leq t \leq T$

Measurements: $y = (y(t_1), y(t_2) \ldots y(t_M))$

Model:

- Process model: $\dot{x} = u$
- High forces have lower probability: $P(u) \sim \exp\left[-\frac{u^2}{2\sigma_u^2}\right]$

- Observation model: $y = x + v$
- Gaussian measurement error: $P(v) \sim \exp\left[-\frac{v^2}{2\sigma_v^2}\right]$
Optimal dynamic motion estimation (2)

Probability that $x(t)$ is the correct trajectory, given data $y$:

$$P(x(t) \mid y_1, y_2 \ldots y_M) = \underbrace{P(y_1, y_2 \ldots y_M \mid x(t))}_{\text{likelihood}} \cdot \underbrace{P(x(t))}_{\text{prior}}$$

$$= \underbrace{P(y_1 \mid x(t_1)) \cdot P(y_2 \mid x(t_2)) \ldots P(y_M \mid x(t_M))}_{\text{data likelihood}} \cdot \underbrace{P(\ddot{x}(t_1)) \cdot P(\ddot{x}(t_2)) \ldots P(\ddot{x}(t_K))}_{\text{state transition}}$$

$$\sim \exp \left[ -\frac{(y_1 - x(t_1))^2}{2\sigma_v^2} \right] \exp \left[ -\frac{(y_2 - x(t_2))^2}{2\sigma_v^2} \right] \ldots \exp \left[ -\frac{\ddot{x}(t_1)^2}{2\sigma_u^2} \right] \exp \left[ -\frac{\ddot{x}(t_2)^2}{2\sigma_u^2} \right] \ldots$$

taking $K \to \infty$ and factoring out constants:

$$\sim \exp \left[ -\sum_{i=1}^{M} (y_i - x(t_i))^2 - p \int_0^T \dddot{x}(t)^2 \, dt \right]$$

with $p = \frac{\sigma_v^2}{\sigma_u^2 \cdot T}$.
The optimal estimate is the trajectory $x(t)$ which minimizes:

$$\sum_{i=1}^{M} (y_i - x(t_i))^2 + p \int_0^T \ddot{x}(t)^2 \, dt$$

This is a trajectory optimization problem. The solution is a **cubic smoothing spline**.

Which is (apart from boundary conditions) equivalent to a double (forward & backward) 2$^{nd}$ order low pass Butterworth filter.
General, state space formulation

Find state trajectory \( x(t) \) and control trajectory \( u(t) \) that minimize:

\[
J = \sum_{i=1}^{M} \left( y_i - g\left( x(t_i) \right) \right)^2 + p \int_{0}^{T} u(t)^2 \, dt
\]

Subject to the process model: \( f(x, \dot{x}, u) = 0 \)

Observation model: \( y = g(x) + v \) (sensor fusion)

Cost function \( J \): derived from Gaussian \( v \) and \( u \) & maximizing \( P(H|E) \)

Solution methods for trajectory optimization:
- Shooting (repeated simulation while iterating \( u(t) \))
- Multiple Shooting (Mombaur)
- Direct Collocation (Ackermann & van den Bogert, J Biomech 2010)
Control / effort cost term

Traditionally based on physics or physiology:
- Electric motors: heat ~ sum $u^2$
- Muscles: fatigue ~ sum $u^3$

Probabilistic interpretation:
- Gaussian prior: minimize sum $u^2$
- Generalized Gaussian: minimize sum $u^k$
- Uniform: minimize max($u$)

$$P(u) \sim e^{-\left(\frac{u}{\sigma}\right)^k}$$

$$\text{cost} \sim \sum u^k$$

Uniform prior probability:
$$k = \infty$$

$$\text{cost} = \max(u)$$
Implicit musculoskeletal dynamics

\[ \mathbf{x} = \begin{pmatrix} \mathbf{q} \\ \mathbf{v} \\ \mathbf{s} \\ \mathbf{a} \end{pmatrix} \]

- generalized coordinates
- generalized velocities
- projected fiber lengths
- muscle activation states

\[ \begin{align*}
\dot{\mathbf{q}} - \mathbf{v} \\
M(\mathbf{q})\dot{\mathbf{v}} + \mathbf{B}(\mathbf{q}, \mathbf{v}) + \left( \frac{\partial \mathbf{L}_M}{\partial \mathbf{q}} \right)^T \mathbf{F}_{\text{SEE}} (\mathbf{L}_M(\mathbf{q}) - \mathbf{s}) \\
\mathbf{F}_{\text{SEE}}(\mathbf{L}_M(\mathbf{q}) - \mathbf{s}) - \left( a F_{\text{max}} \mathbf{f}_{FL}(L_{CE}) \mathbf{f}_{FV}(\dot{L}_{CE}) - f_{\text{PEE}}(L_{CE}) \right) \cos \phi \\
\dot{\mathbf{a}} - (\mathbf{u} - \mathbf{a}) \left( \frac{\mathbf{u}}{T_{\text{act}}} + \frac{1 - \mathbf{u}}{T_{\text{deact}}} \right)
\end{align*} \]

\[ f(\mathbf{x}, \dot{\mathbf{x}}, \mathbf{u}) = 0 \]

\[ L_{CE} = \sqrt{s^2 + L_{\text{CE ref}}^2 \sin^2 \phi_{\text{ref}}} \]

\[ \cos \phi = s / L_{CE} \]

van den Bogert et al., Procedia IUTAM 2011
downhill skiing
state estimation from low quality video data
Landing movement in downhill skiing

Data:
- low resolution video
- 2 panning cameras
- manual digitization
- 3D calibration in each frame

Nachbauer et al., J Appl Biomech 1996
Model and optimization

Process model:
- planar 9-DOF skeleton
- 9-segment flexible skis
- 16 muscles
- ski boot stiffness
- ski-ground contact & friction

\[ \mathbf{x} = (x_1, x_2, \ldots, x_{82}) \]

\[ f(\mathbf{x}, \dot{\mathbf{x}}, \mathbf{u}) = 0 \]

Observation model:
\[ \mathbf{g} (\mathbf{x}) = (q_1, q_2, \ldots, q_9) + \mathbf{v} \]

Cost function:
\[ J = \sum_{i=1}^{M} \left( y_i - g(x(t_i)) \right)^2 + p \int_{0}^{T} u(t)^2 \, dt \]

Solution:
Kinematics of the landing movement

trajectory optimization preserves high frequencies at impact

Other variables

Ground reaction force

Left knee
Right knee

Muscle controls (u)

Knee ligament forces

Left knee
Right knee
walking and running
state estimation from IMU data
Introduction

- Inertial Measurement Unit (IMU)
  - 3D accelerometer, 3D rate gyro, 3D magnetometer
  - less expensive than optical motion capture
  - not restricted to laboratory environment

- Kinematic analysis
  - sensor orientation from Kalman filter
  - human inverse kinematics (*Roetenberg et al., 2009*)

- Inverse dynamic analysis and muscle forces
  - position drift → no registration with force plate possible
  - GRF prediction method (*Karatsidis et al., 2019*)
    - inverse kinematics, inverse dynamics, static optimization
    - complete kinematic data needed

$36 - www.sparkfun.com

$12,000 - www.xsens.com
Trajectory optimization approach

- Model of musculoskeletal dynamics: $f(x, \dot{x}, u) = 0$
- Find periodic trajectories $x(t)$ and $u(t)$ to minimize:

$$F(x(t), u(t)) = \frac{1}{T} \int_{0}^{T} \left[ \frac{1}{N_{\text{sensors}}} \sum_{i=1}^{N_{\text{sensors}}} \left( \frac{s_{i}(x(t)) - m_{i}(t)}{\sigma_{i}} \right)^2 + \frac{W}{N_{\text{muscles}}} \sum_{i=1}^{N_{\text{muscles}}} u_{i}(t)^2 \right] dt$$

- Potential advantages
  - simultaneous estimation of kinematics, forces, muscle states
  - solutions are dynamically consistent (obey physics and muscle physiology)
  - can track raw unfiltered sensor data
  - can work with incomplete data (and overcomplete data)
Model

- Musculoskeletal dynamics model \( f(x, \dot{x}, u) = 0 \)
  - planar, 9 DOF
  - 16 Hil-based muscles
  - 2 ground contact points on each foot
  - 66 state variables \( x \), 16 control inputs \( u \)

Sensor models

- gyro: \( g_s := \omega_s \)
- accelerometer: \( a_s := R^{B_sG} \left( a^G_{G|B_s} - g \right) + \begin{bmatrix} -\omega_s^2 & -\dot{\omega}_s \\ \dot{\omega}_s & -\omega_s^2 \end{bmatrix} p^{B_s} \)

van den Bogert, Read, Nigg, J Biomech 1996
Evaluation study (J Biomech, in press)

- 10 subjects, 6 speeds (0.9 - 4.9 m/s)
- 10 trials at each speed
- 7 IMUs (sacrum, lateral thigh & shank, dorsal foot)
- Optical motion capture (OMC), force plates
  - 2D joint angles and moments (Winter, 2009)
- Ensemble averaging of data
  - 100 points in average gait cycle, $m \pm \sigma$
- Trajectory optimization
- Comparison IMU vs. OMC results
  - correlation coefficients
  - RMS differences

\[
\text{deviation from measurements } m(t), \text{ using sensor model } s(x)
\]
Sensor signal tracking (running, one subject)

Soft tissue artifacts are rejected.

Impact peaks are tracked.
Joint angles and moments (walking, one subject)

Effect of walking speed is correctly estimated
Joint angles and moments (running, one subject)

- Hip flexion
- Knee extension
- Ankle dorsiflexion

Ankle moment rises too early.
Correlations (all data, 10 x 6 x 100 points)

systematic overestimation
Discussion

- Excellent correlation between IMU and OMC kinematics/kinetics ($\rho > 0.9$)
- Estimates of muscle forces and contraction/activation states
  - consistent with typical EMG
  - metabolic energy cost can be calculated from muscle state trajectories

Limitations

- computation speed: ~50 min for each of the 60 optimizations
- ankle moment overestimation, likely due to rigid foot model
- 2D analysis
- muscle load sharing is guided by optimization objective
Future work

- Fewer sensors:
  - minimal sensor set?
- More sensors:
  - EMG, foot pressure
- Self-calibration
- Real time
- 3D analysis
- Validation for clinical and sports applications
Muscle forces

- Iliopsoas: 400 N
- Gluteus: 200 N
- Vastus: 1,500 N
- Gastrocnemius: 1,000 N
- Hamstrings: 150 N
- Rectus: 200 N
- Soleus: 1,500 N
- Tibialis Anterior: 600 N
3D results

Joint Angles in °

Joint Moments in N m

GRFs in BW

Hip Flex.

Hip Adduc.

Hip Rot.

Knee Flex.

Ankle Dorsiflex.

Gait Cycle in %

Gait Cycle in %
Bayes' theorem

\[
P(A | B) \cdot P(B) = P(B | A) \cdot P(A) \implies P(A | B) = \frac{P(B | A) \cdot P(A)}{P(B)}
\]